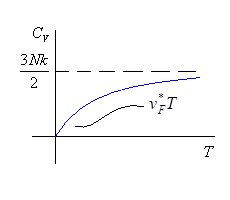
**Thermal Equilibrium Properties**

**Heat Capacity**

C­V(T) doesn’t seem to be affected much by the e-e interaction, except to renormalize m again, to m\*, and thereby renormalize the Fermi velocity vF = kF/m:



What role do the plasma oscillations play? Perhaps not too large at low T in any event, since there is a ‘mass’ gap; that is, there is a non-zero energy minimum.

**Compressibility**

We define the following two quantities – at T = 0 – I suppose.



the extra factor of V is defined so that the compressibility so defined is independent of the system size. And P would be -∂F/∂V, but at T = 0, F = E – TS = E. This is necessary since we note that E and V are extrinsic quantities, while P is intrinsic. So we do need the extra V to make K an intrinsic quantity. For instance, the free fermion gas at T = 0 has values of:



The procedure for finding the compressibility may be generalized to arbitrary systems. The total energy of our interacting gas, and chemical potential (which would be EF at T = 0) was (at T = 0):



Performing the operations at the top, we arrive at, for P,



since,



and this implies that:



We may use the expression for EGS discussed before and with it calculate 1/K. Another consideration is to recognize from Seitz’s theorem that:



and so it follows from this that:



We’ll use *this* expression to calculate the compressibility, and evaluate K using the HF approximation. HF tells us that,



we write kF in terms of n0, take the requisite derivative,



Can write this as:



Now we’ll note that 1/me2 = a0, the Bohr radius, in our units. So we can write this as:



But from excitations file, we said,



So this is:



which comes out to, roughly,



and we see that the compressibility is increased. That’s interesting, given the e-e repulsion (but there is also positive jelly) And we see evidence of a transition at rs = 6.02 to some other state (phase transitions are heralded when susceptibilities blow up, just like analogously happens with the Van der Waals gas) But we also know that it’s unreliable to treat the chemical potential as being the sum of just those two terms. There is yet another way to calculate this via the ever useful ε (dielectric function). Nozieres (1964) demonstrated that:



We can rewrite this in terms of Kf. Note that (qTF defined in the susceptibility file):



So we have:



In the RPA limit, for instance, we’d have:



Let’s use Hubbard’s result instead (see non-equilibrium properties folder/Electric Susceptibility):



and so we end up with:



So then comparing to the compressibility theorem, we have,



From the susceptibility file, we have:



and since a0 = 1/me2 in our units (‘faux’ Natural + Gausian), we can write:



More concretely,



Interestingly, due to the inexactness of the ε calculation in the HF, Hubbard, SS, etc., calculations these two aforementioned methods (EGS vs. ε method) gives slightly different results for K. But in all cases, there is a critical density, rs = rc, at which K → ∞, and below which the K goes negative. This is an actual prediction that the GS of the electron gas is unstable – it would literally predict that an increase in P results in an increase in V. In fact there does exist a critical density at which the electron gas GS shifts (a quantum phase transition) to an insulating state – the Wigner lattice.